Ch 9.3 Geometric Sequences and Series Lessons

SECTION 9.3 GEOMETRIC SEQUENCES AND SERIES

SKILLS OBJECTIVES

- Recognize a geometric sequence.
- Find the general, nth term of a geometric sequence.
- Evaluate a finite geometric series.
- Evaluate an infinite geometric series, if it exists.
- Use geometric sequences and series to model real-world problems.

CONCEPTUAL OBJECTIVES

- Understand the difference between a geometric sequence and a geometric series.
- Distinguish between an arithmetic sequence and a geometric sequence.
- Understand why it is not possible to evaluate all infinite geometric series.

Geometric Sequences

In Section 9.2, we discussed arithmetic sequences, where successive terms had a common difference. In other words, each term was found by adding the same constant to the previous term. In this section we discuss geometric sequences, where successive terms have a common ratio. In other words, each term is found by multiplying the previous term by the same constant. The sequence 4, 12, 36, 108 .... is geometric because each successive term is found by multiplying the previous term by 3.

DEFINITION Geometric Sequences

A sequence is **geometric** if each term in the sequence is found by multiplying the previous term by a number r, so that $a_{n+1} = r \cdot a_n$. Because $\frac{a_{n+1}}{a_n} = r$, the number r is called the **common ratio**.

DEFINITION Geometric Sequences

A sequence is geometric if each term in the sequence is found by multiplying the previous term by a number r, so that $a_{n+1} = r \bullet a_n$. Because

$$\frac{a_{n+1}}{a_n} = r$$

, the number r is called the common ratio.

EXAMPLE 1 Identifying the Common Ratio in Geometric Sequences

Find the common ratio for each of the geometric sequences.

a. 5, 20, 80, 320,...

Solution (a):

Label the terms. $a_1 = 5, a_2 = 20, a_3 = 80, a_4 = 320, \ldots$

Find the ratio $r = \frac{a_{n+1}}{a_n}$.

$$r = \frac{a_2}{a_1} = \frac{20}{5} = 4$$

$$r = \frac{a_3}{a_2} = \frac{80}{20} = 4$$

$$r = \frac{a_4}{a_3} = \frac{320}{80} = 4$$

$\boxed{\text{The common ratio is 4.}}$

Find the common ratio for each of the geometric sequences.

b. $1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots$

Solution (b):

Label the terms. $a_1 = 1, a_2 = -\frac{1}{2}, a_3 = \frac{1}{4}, a_4 = -\frac{1}{8}, \ldots$

Find the ratio $r = \frac{a_{n+1}}{a_n}$.

$$r = \frac{a_2}{a_1} = \frac{-1/2}{1} = -\frac{1}{2}$$

$$r = \frac{a_3}{a_2} = \frac{1/4}{-1/2} = -\frac{1}{2}$$

$$r = \frac{a_4}{a_3} = \frac{-1/8}{1/4} = -\frac{1}{2}$$

$\boxed{\text{The common ratio is } -\tfrac{1}{2}.}$

c. \$5.000, \$5,500, \$6,050, \$6, 655, ...

Solution (c):

Label the terms. $a_1 = \$5{,}000,\ a_2 = \$5{,}500,\ a_3 = \$6{,}050,\ a_4 = \$6{,}655, \ldots$

Find the ratio $r = \dfrac{a_{n+1}}{a_n}$.

$$r = \frac{a_2}{a_1} = \frac{\$5{,}500}{\$5{,}000} = 1.1$$

$$r = \frac{a_3}{a_2} = \frac{\$6{,}050}{\$5{,}500} = 1.1$$

$$r = \frac{a_4}{a_3} = \frac{\$6{,}655}{\$6{,}050} = 1.1$$

The common ratio is 1.1.

• ***YOUR TURN*** ***Find the common ratio of each geometric series.***

a. 1, -3.9. -27....

a. solution: -3

b. 320, 80, 20, 5,...

b. solution $\frac{1}{4}$ or 0.25

The General (nth) Term of a Geometric Sequence

To find a formula for the general, or nth, term of a geometric sequence, write out the first several terms and look for a pattern.

First term, $n = 1$. $\quad a_1$

Second term, $n = 2$. $\quad a_2 = a_1 \cdot r$

Third term, $n = 3$. $\quad a_3 = a_2 \cdot r = (a_1 \cdot r) \cdot r = a_1 \cdot r^2$

Fourth term, $n = 4$. $\quad a_4 = a_3 \cdot r = (a_1 \cdot r^2) \cdot r = a_1 \cdot r^3$

In general, the nth term is given by $a_n = a_1 \cdot r^{n-1}$.

THE *n*TH TERM OF A GEOMETRIC SEQUENCE

The ***n*th term** of a geometric sequence with common ratio r is given by

$$a_n = a_1 \cdot r^{n-1} \text{ for } n \geq 1$$

THE nTH TERM OF A GEOMETRIC SEQUENCE

The nth term of a geometric sequence with common ratio r is given by

$$a_n = a_1 \bullet r^{n-1} \quad \text{for n} \geq 1$$ or by

$$a_{n+1} = a_1 \bullet r^{n} \quad \text{for n} \geq 0$$

EXAMPLE 2 *Finding the nth Term of a Geometric Sequence*

Find the 7th term of the sequence 2, 10, 50, 250

Solution:

Identify the common ratio. $r = \frac{10}{2} = \frac{50}{10} = \frac{250}{50} = 5$

Identify the first ($n = 1$) term. $a_1 = 2$

Substitute $a_1 = 2$ and $r = 5$ into $a_n = a_1 \cdot r^{n-1}$. $a_n = 2 \cdot 5^{n-1}$

Substitute $n = 7$ into $a_n = 2 \cdot 5^{n-1}$. $a_7 = 2 \cdot 5^{7-1} = 2 \cdot 5^6 = 31{,}250$

The 7th term of the geometric sequence is 31,250.

Technology Tip

Use seq to find the nth term of the sequence by setting the initial index value equal to the final index value. To find the 7th term of the geometric sequence $a_n = 2 \cdot 5^{n-1}$, press

2nd LIST ▶ OPS ▼ 5:seq(
ENTER 2 x 5 ^ (ALPHA
N – 1) , ALPHA N ,
7 , 7 , 1) ENTER

```
seq(2*5^(N-1),N,
7,7,1)
                {31250}
```

• ***YOUR TURN*** ***Find the 8th term of the sequence 3, 12, 48, 192***

•Answer: 49,152

EXAMPLE 3 ***Finding the Geometric Sequence***

Find the geometric sequence whose 5th term is 0.01 and whose common ratio is 0.1.

Solution:

Label the common ratio and 5th term. $a_5 = 0.01$ and $r = 0.1$

Substitute $a_5 = 0.01$, $n = 5$, and $r = 0.1$ into $a_n = a_1 \cdot r^{n-1}$. $0.01 = a_1 \cdot (0.1)^{5-1}$

Solve for a_1. $a_1 = \frac{0.01}{(0.1)^4} = \frac{0.01}{0.0001} = 100$

The geometric sequence that starts at 100 and has a common ratio of 0.1 is 100, 10, 1, 0.1, 0.01, ...

• ***YOUR TURN***

Find the geometric sequence whose 4th term is 3 and whose common ratio is $\frac{1}{3}$.

•Answer: 81, 27, 9, 3, 1....

Geometric Series

The sum of the terms of a geometric sequence is called a **geometric series**.

$$a_1 + a_1 \cdot r + a_1 \cdot r^2 + a_1 \cdot r^3 + \cdots$$

If we only sum the first n terms of a geometric sequence, the result is a **finite geometric series** given by

$$S_n = a_1 + a_1 \cdot r + a_1 \cdot r^2 + a_1 \cdot r^3 + \cdots + a_1 \cdot r^{n-1}$$

To develop a formula for the nth partial sum, we multiply the above equation by r:

$$r \cdot S_n = a_1 \cdot r + a_1 \cdot r^2 + a_1 \cdot r^3 + \cdots + a_1 \cdot r^{n-1} + a_1 \cdot r^n$$

Subtracting the second equation from the first equation, we find that all of the terms on the right side drop out except the first term in the first equation and the last term in the second equation:

$$\begin{array}{rl} S_n = & a_1 + a_1 \cdot r + a_1 \cdot r^2 + \cdots + a_1 r^{n-1} \\ -rS_n = & \quad - a_1 \cdot r - a_1 \cdot r^2 - \cdots - a_1 r^{n-1} - a_1 r^n \\ \hline S_n - rS_n = & a_1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad - a_1 r^n \end{array}$$

Continued on next page ….

Factor the S, out of the left side and the a_1 , out of the right side:

$$S_n(1 - r) = a_1(1 - r^n)$$

Divide both sides by (1- r), assuming $r \neq 1$.

The result is a general formula for the sum of a finite geometric series:

$$S_n = a_1 \frac{(1 - r^n)}{(1 - r)} \quad r \neq 1$$

$$S_n = \sum_{k=1}^{n} a_1 \bullet r^{k-1} = a_1 + a_1 \bullet r + a_1 \bullet r^2 + a_1 \bullet r^3 + \bullet\bullet\bullet + a_1 \bullet r^{n-1}$$

EVALUATING A FINITE GEOMETRIC SERIES

The sum of the first n terms of a geometric sequence, called a **finite geometric series**, is given by the formula

$$S_n = a_1 \frac{(1 - r^n)}{(1 - r)} \qquad r \neq 1$$

It is important to note that a finite geometric series can also be written in sigma (summation) notation:

$$S_n = \sum_{k=1}^{n} a_1 \cdot r^{k-1} = a_1 + a_1 \cdot r + a_1 \cdot r^2 + a_1 \cdot r^3 + \cdots + a_1 \cdot r^{n-1}$$

Study Tip

The underscript k = 1 applies only when the summation starts at the a_1 term.
It is important to note which term is the starting term.

EXAMPLE 4 Evaluating a Finite Geometric Series

Evaluate the finite geometric series.

a. $\sum_{k=1}^{13} 3 \cdot (0.4)^{k-1}$

Solution (a):

Identify a_1, n, and r. $\qquad a_1 = 3, n = 13$, and $r = 0.4$

Substitute $a_1 = 3$, $n = 13$, and $r = 0.4$ into $s_n = a_1 \frac{(1 - r^n)}{(1 - r)}$. $\qquad S_{13} = 3\frac{\left(1 - 0.4^{13}\right)}{(1 - 0.4)}$

Simplify. $\qquad \boxed{S_{13} \approx 4.99997}$

Technology Tip

a. To find the sum of the series $\sum_{k=1}^{13} 3 \cdot (0.4)^{k-1}$, press

[2nd] [LIST] [▶] [MATH] [▼]
[5:sum(] [ENTER] [2nd] [LIST] [▶]
[OPS] [▼] [5:seq(] [ENTER] 3 [x]
[(] 0.4 [)] [^] [(] [ALPHA] [K] [−]
1 [)] [,] [ALPHA] [K] [,] 1 [,] 13 [,]
1 [)] [)] [ENTER]

```
sum(seq(3*(0.4)^
(K-1),K,1,13,1))
        4.999966446
```

b. The first nine terms of the series 1 + 2 + 4 + 8 + 16 + 32 + 64 +

Solution (b):

Identify the first term and common ratio.	$a_1 = 1$ and $r = 2$
Substitute $a_1 = 1$ and $r = 2$ into $S_n = a_1 \dfrac{(1 - r^n)}{(1 - r)}$.	$S_n = \dfrac{(1 - 2^n)}{(1 - 2)}$
To sum the first nine terms, let $n = 9$.	$S_9 = \dfrac{(1 - 2^9)}{(1 - 2)}$
Simplify.	$\boxed{S_9 = 511}$

b. To find the sum of the first nine terms of the series $1 + 2 + 4 + 8 + 16 + 32 + 64 + \cdots$, press

[2nd] [LIST] [▶] [MATH] [▼]
[5:sum(] [ENTER] [2nd] [LIST] [▶]
[OPS] [▼] [5:seq(] [ENTER] 2 [^]
[(] [ALPHA] [K] [−] 1 [)] [,]
[ALPHA] [K] [,] 1 [,] 9 [,] 1 [)] [)]
[ENTER]

```
sum(seq(2^(K-1),
K,1,9,1))
              511
```

The sum of an infinite geometric sequence is called an **infinite geometric series**. Some infinite geometric series converge (yield a finite sum) and some diverge (do not have a finite sum). For example,

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots + \frac{1}{2^n} + \cdots = 1 \quad \text{(converges)}$$

$$2 + 4 + 8 + 16 + 32 + \cdots + 2^n + \cdots \quad \text{(diverges)}$$

For infinite geometric series that converge, the partial sum S, approaches a single number as n gets large. The formula used to evaluate a finite geometric series

$$S_n = a_1 \frac{(1 - r^n)}{(1 - r)}$$

can be extended to an infinite geometric series for certain values of,:

If $|r| < 1$, then when r is raised to a power, it continues to get smaller, approaching 0.

For those values of r, infinite geometric series converges to a finite sum.

$$\text{Let } n \to \infty \text{; then } a_1 \frac{(1 - r^n)}{(1 - r)} \to a_1 \frac{(1 - 0)}{(1 - r)} = a_1 \frac{1}{1 - r}, \text{ if } |r| < 1.$$

EVALUATING AN INFINITE GEOMETRIC SERIES

The **sum of an infinite geometric series** is given by the formula

$$\sum_{n=0}^{\infty} a_1 \cdot r^n = a_1 \frac{1}{(1 - r)} \qquad |r| < 1$$

EVALUATING AN INFINITE GEOMETRIC SERIES

The sum of an infinite geometric series is given by the formula

$$\sum_{n=0}^{\infty} a_1 \bullet r^n = a_1 \frac{1}{(1-r)} \qquad |r|<1$$

Study Tip

The formula used to evaluate an infinite geometric series is: $\dfrac{\text{First term}}{1-\text{Ratio}}$

EXAMPLE 5 ***Determining Whether the Sum of an Infinite Series Exists***

Determine whether the sum exists for each of the geometric series.

$a.\quad 3+15+75+375+\bullet\bullet\bullet$

Solution (a):

Identify the common ratio. $r = 5$

Since 5 is greater than 1, the $\boxed{\text{sum does not exist}}$. $r = 5 > 1$

Determine whether the sum exists for each of the geometric series.

b. $8 + 4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$

Solution (b):

Identify the common ratio. $r = \frac{1}{2}$

Since $\frac{1}{2}$ is less than 1, the $\boxed{\text{sum exists}}$. $r = \frac{1}{2} < 1$

• YOUR TURN
Determine whether the sum exists for the following geometric series.

a. $81, 9, 1, \frac{1}{9}, \ldots$

b. $1, 5, 25, 125, \ldots$

Answer: a. yes b. no

Do you expect $\frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \frac{1}{64} + \cdots$ and $\frac{1}{4} - \frac{1}{12} + \frac{1}{36} - \frac{1}{64} + \cdots$
+ + + + ... and — + — + to sum to the same number?

The answer is no, because the second series is an alternating series and terms are both added and subtracted.

Hence, we would expect the second series to sum to a smaller number than the first series sums to.

EXAMPLE 6 ***Evaluating an Infinite Geometric Series***

Evaluate each infinite geometric series.

a. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

b. $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$

Solution (a):

Identify the first term and the common ratio. $a_1 = 1 \qquad r = \dfrac{1}{3}$

Since $|r| = \left|\frac{1}{3}\right| < 1$, the sum of the series exists.

Substitute $a_1 = 1$ and $r = \frac{1}{3}$ into $\sum_{n=0}^{\infty} a_1 \cdot r^n = a_1 \dfrac{1}{(1 - r)}$. $\dfrac{1}{1 - 1/3}$

Simplify. $= \dfrac{1}{2/3} = \dfrac{3}{2}$

$$\boxed{1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots = \frac{3}{2}}$$

Study Tip

$$\sum_{n=0}^{\infty} a_1 \cdot r^n = \frac{\text{First term}}{1 - \text{Ratio}} = \frac{1}{1 - 1/3}$$

Solution (b):

Identify the first term and the common ratio. $a_1 = 1 \qquad r = -\frac{1}{3}$

Since $|r| = |-\frac{1}{3}| < 1$, the sum of the series exists.

Substitute $a_1 = 1$ and $r = -\frac{1}{3}$ into $\sum_{n=0}^{\infty} a_1 \cdot r^n = a_1 \frac{1}{(1-r)}$.

Simplify. $= \frac{1}{1-(-1/3)} = \frac{1}{1+(1/3)} = \frac{1}{4/3} = \frac{3}{4}$

$$\boxed{1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots = \frac{3}{4}}$$

Study Tip

$$\sum_{n=0}^{\infty} a_1 \cdot r^n = \frac{\text{First term}}{1 - \text{Ratio}} = \frac{1}{1-(-1/3)}$$

Notice that the alternating series summed to $\frac{3}{4}$, whereas the positive series summed to $\frac{3}{2}$.

• ***YOUR TURN*** ***Find the sum of each infinite geometric series.***

a. $\frac{1}{4} + \frac{1}{12} + \frac{1}{36} + \frac{1}{64} + \cdots$

b. $\frac{1}{4} - \frac{1}{12} + \frac{1}{36} - \frac{1}{64} + \cdots$

■ **Answer: a.** $\frac{3}{8}$ **b.** $\frac{3}{16}$

It is important to note the restriction on the common ratio r.

The absolute value of the common ratio has to be strictly less than 1 for an infinite geometric series to converge.

Otherwise the infinite geometric series diverges.

EXAMPLE 7 ***Evaluating an Infinite Geometric Series***

Evaluate the infinite geometric series, if possible.

a. $\sum_{n=0}^{\infty} 2\left(-\frac{1}{4}\right)^n$

Solution (a):

Identify a_1 and r.

$$\sum_{n=0}^{\infty} 2\left(-\frac{1}{4}\right)^n = \underbrace{2}_{a_1} - \underbrace{\frac{1}{2} + \overbrace{\frac{1}{8} - \frac{1}{32}}^{r=-\frac{1}{4}}}_{r=-\frac{1}{4}} + \underbrace{\frac{1}{128} - \cdots}_{r=-\frac{1}{4}}$$

Since $|r| = |-\frac{1}{4}| = \frac{1}{4} < 1$, the infinite geometric series converges.

$$\sum_{n=0}^{\infty} a_1 \cdot r^n = \frac{a_1}{(1-r)}$$

Let $a_1 = 2$ and $r = -\frac{1}{4}$.

$$= \frac{2}{[1-(-1/4)]}$$

Simplify.

$$= \frac{2}{1+1/4} = \frac{2}{5/4} = \frac{8}{5}$$

This infinite geometric series converges.

$$\boxed{\sum_{n=0}^{\infty} 2\left(-\frac{1}{4}\right)^n = \frac{8}{5}}$$

b. $\sum_{n=1}^{\infty} 3 \cdot (2)^{n-1}$

Solution (b):

Identify a_1 and r.

$$\sum_{n=1}^{\infty} 3 \cdot (2)^{n-1} = \underbrace{3}_{a_1} + \overbrace{6 + 12}^{r=2} + \underbrace{24 + 48}_{r=2} + \cdots$$

$\boxed{\text{Since } r = 2 > 1\text{, this infinite geometric series diverges.}}$

Applications

Suppose you are given a job offer with a guaranteed percentage raise per year. What will your annual salary be 10 years from now? That answer can be obtained using a geometric sequence. Suppose you want to make voluntary contributions to a retirement account directly debited from your paycheck every month. Suppose the account earns a fixed percentage rate:

1. How much will you have in 30 years if you deposit $50 a month?
2. What is the difference in the total you will have in 30 years if you deposit $100 a month instead?

These important questions about your personal finances can be answered using geometric sequences and series.

EXAMPLE B ***Future Salary: Geometric Sequence***

Suppose you are offered a job as an event planner for the PGA Tour. The starting salary is $45,000, and employees are given a 5% raise per year. What will your annual salary be during the 10th year with the PGA Tour?

Solution:

Every year the salary is 5% more than the previous year.

Label the year 1 salary. $a_1 = 45{,}000$

Calculate the year 2 salary. $a_2 = 1.05 \cdot a_1$

Calculate the year 3 salary. $a_3 = 1.05 \cdot a_2$

$= 1.05(1.05 \cdot a_1) = (1.05)^2 a_1$

Calculate the year 4 salary. $a_4 = 1.05 \cdot a_3$

$= 1.05(1.05)^2 a_1 = (1.05)^3 a_1$

Identify the year n salary. $a_n = 1.05^{n-1} a_1$

Substitute $n = 10$ and $a_1 = 45{,}000$. $a_{10} = (1.05)^9 \cdot 45{,}000$

Simplify. $a_{10} \approx 69{,}809.77$

$\boxed{\text{During your 10th year with the company your salary will be \$69,809.77.}}$

Study Tip

$$\sum_{n=1}^{10} 45{,}000(1.05)^9 \cong 69{,}809.77$$

• YOUR TURN Suppose you are offered a job with AT&T at $37,000 per year with a guaranteed raise of 4% after every year.
What will your annual salary be after 15 years with the company?

• Answer: $64,072.03

EXAMPLE 9 ***Savings Growth: Geometric Series***

Karen has maintained acrylic nails by paying for them with money earned from a part-time job. After hearing a lecture from her economics professor on the importance of investing early in life, she decides to remove the acrylic nails, which cost $50 per month, and do her own manicures. She has that S50 automatically debited from her checking account on the first of every month and put into a money market account that receives 3% interest compounded monthly. What will the balance be in the money market account exactly 2 years from the day of her initial $50 deposit?

Solution:

Recall the compound interest formula. $A = P\left(1 + \frac{r}{n}\right)^{nt}$

Substitute $r = 0.03$ and $n = 12$ into the compound interest formula. $A = P\left(1 + \frac{0.03}{12}\right)^{12t}$

$= P(1.0025)^{12t}$

Let $t = \frac{n}{12}$, where n is the number of months of the investment: $A_n = P(1.0025)^n$

The first deposit of $50 will gain interest for 24 months. $A_{24} = 50(1.0025)^{24}$

The second deposit of $50 will gain interest for 23 months. $A_{23} = 50(1.0025)^{23}$

The third deposit of $50 will gain interest for 22 months. $A_{22} = 50(1.0025)^{22}$

The last deposit of $50 will gain interest for 1 month. $A_1 = 50(1.0025)^1$

Sum the amounts accrued from the 24 deposits.

$$A_1 + A_2 + \cdots + A_{24} = 50(1.0025) + 50(1.0025)^2 + 50(1.0025)^3 + \cdots + 50(1.0025)^{24}$$

Identify the first term and common ratio. $a_1 = 50(1.0025)$ and $r = 1.0025$

Sum the first n terms of a geometric series. $S_n = a_1\frac{(1 - r^n)}{(1 - r)}$

Substitute $n = 24$, $a_1 = 50(1.0025)$, and $r = 1.0025$. $S_{24} = 50(1.0025)\frac{(1 - 1.0025^{24})}{(1 - 1.0025)}$

Simplify. $S_{24} \approx 1238.23$

Karen will have $1,238.23 saved in her money market account in 2 years.

Technology Tip

Use a calculator to find

$$S_{24} = 50(1.0025)\frac{(1 - 1.0025^{24})}{(1 - 1.0025)}.$$

Scientific calculators:

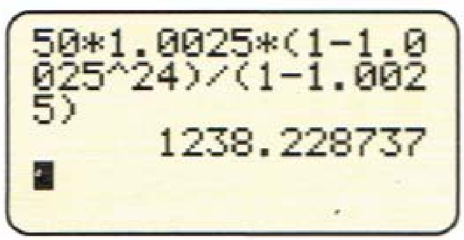
Press	*Display*
50 $\boxed{\text{x}}$ 1.0025 $\boxed{\text{x}}$	1238.23
$\boxed{(}$ 1 $\boxed{-}$ 1.0025	
$\boxed{x^y}$ 24 $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$	
1 $\boxed{-}$ 1.0025 $\boxed{)}$ $\boxed{=}$	

Graphing calculators:

50 $\boxed{\text{x}}$ 1.0025 $\boxed{\text{x}}$ $\boxed{(}$ 1 $\boxed{-}$ 1.0025

$\boxed{\wedge}$ 24 $\boxed{)}$ $\boxed{\div}$ $\boxed{(}$ 1 $\boxed{-}$ 1.0025 $\boxed{)}$

$\boxed{\text{ENTER}}$



• ***YOUR TURN***

Repeat Example 9 with Karen putting $100 (instead of $50) in the same money market. Assume she does this for 4 years (instead of 2 years).

Answer: $5.105.85

SECTION 9.3 SUMMARY

Section 9.3 Summary

In this section, we discussed geometric sequences, in which each successive term is found by multiplying the previous term by a constant, so that $a_{n+1} = r \bullet a_n$.

That constant, **r**, is called the common ratio.

The n[th] term of a geometric sequence is given by

$$a_n = a_1 \bullet r^{n-1} \quad \text{for n} \geq 1$$ or by

$$a_{n+1} = a_1 \bullet r^n \quad \text{for n} \geq 0$$

The sum of the terms of a geometric sequence is called a geometric series.

Finite geometric series converge to a number.

Infinite geometric series converge to a number if the absolute value of the common ratio is less than 1.

If the absolute value of the common ratio is greater than or equal to 1, the infinite geometric series diverges and the sum does not exist. Many real-world applications involve geometric sequences and series, such as growth of salaries and annuities through percentage increases.

Section 9.3 Summary continued

Finite geometric series:

$$S_n = a_1 \frac{\left(1-r^n\right)}{\left(1-r\right)} \quad r \neq 1$$

$$S_n = \sum_{k=1}^{n} a_1 \bullet r^{k-1} = a_1 + a_1 \bullet r + a_1 \bullet r^2 + a_1 \bullet r^3 + \bullet\bullet\bullet + a_1 \bullet r^{n-1}$$

EVALUATING AN INFINITE GEOMETRIC SERIES

The sum of an infinite geometric series is given by the formula

$$\sum_{n=0}^{\infty} a_1 \bullet r^n = a_1 \frac{1}{\left(1-r\right)} \qquad |r| < 1$$

Next section is 9.3 Exercises